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Accurate phase recovery algorithm in lateral shearing interferometry

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Abstract: A phase recovery algorithm with improved accuracy and efficiency is proposed for test wavefront phase recovery from obtained phase differences in shearing interferometry. The algorithm is based on complete pixel by pixel mapping relationship between test phase and its differences, together with the least square principle. In the algorithm, a special linear equation set is firstly built, from which the test phase can be obtained directly by equation solving. Since the coefficient matrix of the equation is sparse, it is transferred to a small new matrix to reduce memory need and calculation amount. In the meantime, since the matrix is a positive-defined matrix, Choleski's factorzation is adopted for convenient equation solving. Reduced time cost and computer memory need and improved recovery accuracy and efficiency have been demonstrated by computational and experimental testing on the proposed algorithm and its comparison with others. Good noise suppression ability is proved by error propagation characteristic analysis.
Key words: shearing interferometry; phase recovery; algorithm; phase difference

横向剪切干涉测量中准确的相位恢复算法

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摘要: 提出一种剪切干涉测量中实现由相位差分准确高效恢复被测波前相位的算法。该算法是基于被测波前相位与其差分完全的点对点对应关系, 及最小二乘算法原理。首先由被测波前相位与其差分完全的点对点对应关系及最小二乘原理, 建立一特殊的线性等式, 被测相位能通过解此等式直接获得。由于该线性等式的系数矩阵为稀疏矩阵, 能转化为一新的小矩阵, 以降低计算机存储空间和计算量; 同时, 由于该矩阵为正定矩阵, Choleski 因式化分解方法能用来实现该线性等式方便的解。进行了计算机数值分析和相关试验测试, 结果表明: 该恢复算法可行, 且计算精度高、计算复杂性低; 可实现由相位差分准确恢复被测波前相位, 同时具有良好的噪声误差传输特性。
关 键 词: 剪切干涉; 相位差分; 相位恢复; 算法
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1 Introduction

Lateral shearing interference is interference between test wavefront and its lateral sheared copy[1]. In lateral shearing interferometry, no reference wavefront is needed, and the optical path difference (OPD) reacts directly only to the gradient of the test wavefront phase but not the wavefront phase itself. As a result, the density of interference fringes is less sensitive to the test wavefront, so the measurement range can be improved significantly. What's more, since the test wavefront vibration does not cause any change of the OPD, and so does the measurement result, good immunity of the interferometry to environment disturbance can be guaranteed. However, in such interferometry, only differences or slope measurements of the test wavefront phase are obtained from interferogram analysis. To achieve the test wavefront itself, an accurate algorithm should be applied to these measurements for its recovery.

Several algorithms have been proposed for the problem. Integration method[2-3] is a direct recovery algorithm, in which small shear has to be adopted and noise accumulation and error propagation are the very problem for it. Polynomial fitting, such as the Zernike polynomial fitting algorithms[4-7] can reduce the problem due to noise, however in these algorithms, it is necessary to predefine a polynomial representation with specified order number k that is assumed to be in accordance with the unknown test wavefront. As a result of the difficulty in doing this, the recovery error is usually serious. In recent years, Fourier transformation methods have also been attempted to the problem[8-11], however, these methods are somewhat limited due to their inconvenience for use.

Partial differential equation methods, in which no pre-defined function representations are needed, have also been adopted for the wavefront phase recovery problem[12-15]. However when the wavefront phase size is not so small in such methods, computer memory requirement, and calculation cost and speed problems will be faced. Although iterative numerical schemes such as Jacobi and successive over-relaxation have been attempted to deal with the memory and time problems in differential equation solving, the condition that the iteration should be convergent cannot be always satisfied. In fact the convergence rate will be very slow when the size of the coefficient matrix of the equation is large[8].

In the proposed algorithm, based on complete pixel by pixel mapping relationship between test phase and its differences, together with the least square principle, a new equation set has been built for direct test wavefront phase. Since the coefficient matrix of the equation is sparse, it can be transferred to a small new matrix, so that less memory and less time are needed. In the meantime, since the matrix is a positive-defined matrix, Choleski's factorzation method[16] can be adopted for convenient solving and help reduce the calculating amount, computer memory need and improve the algorithm's speed.

Noise impact on recovery result in the algorithm is analyzed to show its good property in avoiding noise propagation or error accumulation.

2 Principle of the recovery algorithm

The test wave-front phase array is expressed discretely as

$$\varphi_{i,j},\ i=1,2,\cdots,N;\ j=1,2,\cdots M;$$

where N, M indicate the size of the array in x and y direction, respectively.

The obtained phase differences corresponding to x and y direction shearing are expressed as following, respectively

$$\varphi^x_{i,j}=1,2,\cdots,N-1;\ j=1,2,\cdots,M;$$

$$\varphi^y_{i,j}=1,2,\cdots,N;\ j=1,2,\cdots,M-1;$$

According to shearing interference principle,

$$\varphi^x_{i,j}=\phi_{i+1,j}-\phi_{i,j},\ i=1,2,\cdots,N-1;\ j=1,2,\cdots,M, \tag{1}$$

and

$$\varphi^y_{i,j} = \phi_{i,j+1} - \phi_{i,j},\ i = 1, 2, \cdots, N;\ j = 1, 2, \cdots, M-1, \tag{2}$$

Suppose the practical phase differences obtained by shearing interferometry are denoted by the following expressions, respectively

$$\tilde{\varphi}^x_{i,j},\ i = 1, 2, \cdots, N-1;\ j = 1, 2, \cdots, M;$$
$$\tilde{\varphi}^y_{i,j},\ i = 1, 2, \cdots, N;\ j = 1, 2, \cdots, M-1;$$

The equation set consists of Eq. (1) and Eq. (2) can be solved to obtain the test wave-front phase array while substituting the practical measurement data $\tilde{\varphi}^x_{i,j}$ and $\tilde{\varphi}^y_{i,j}$ for $\varphi^x_{i,j}$ and $\varphi^y_{i,j}$ respectively. The equation set is an over-determined linear equation set, which can be solved in the least squares sense. The solution $\phi_{i,j}$ is those that minimize the residual square sum

$$f(\phi_{i,j},\ i = 1, 2, \cdots, N,\ j = 1, 2, \cdots, M) = \sum_{i=1}^{N-1}\sum_{j=1}^{M}(\phi_{i+1,j} - \phi_{i,j} - \tilde{\varphi}^x_{i,j})^2 + \sum_{i=1}^{N}\sum_{j=1}^{M-1}(\phi_{i,j+1} - \phi_{i,j} - \tilde{\varphi}^y_{i,j})^2, \tag{3}$$

Set the first derivatives of f relative to $\phi_{i,j}$ to zeros as

$$\frac{\partial}{\partial \phi_{i,j}} f(\phi_{i,j}) = 0, \tag{4}$$

By Eq. (4), a series of equations are obtained as

$$\phi_{i-1,j} + \phi_{i,j-1} - 4\phi_{i,j} + \phi_{i,j+1} + \phi_{i+1,j} = -\tilde{\varphi}^x_{i,j} - \tilde{\varphi}^y_{i,j} + \tilde{\varphi}^x_{i+1,j} + \tilde{\varphi}^y_{i,j+1}$$
$$i = 1, \cdots, N;\ j = 1, \cdots, M, \tag{5}$$

where, the coefficients of terms $\phi_{m,n}$ is set 0 when $m \le 0$ or $m > N$ or $n \le 0$ or $n > M$.

Thus a linear matrix equation is obtained as

$$\boldsymbol{A}\phi = \boldsymbol{p}, \tag{6}$$

where ϕ is a vector whose entries are corresponding to the test wavefront phase to be decided as $\phi = [\phi_{1,1}\ \phi_{1,2}\ \cdots\ \phi_{1,M}\ \phi_{2,1}\ \cdots\ \phi_{2,M}\ \cdots\ \phi_{N-1,M}\ \phi_{N,1}\ \cdots\ \phi_{N,M}]^T$, $\boldsymbol{p}$ is a vector as $\boldsymbol{p} = [p_1 p_2 \cdots p_{NM-1}\ p_{NM}]^T$

$$\boldsymbol{A} = \begin{bmatrix}
-2 & 1 & 0 & \cdots & 0 & 1 & & & & & & & & & & & & & & & \\
1 & -3 & 1 & 0 & \cdots & 0 & 1 & & & & & & & & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & & & & & & & & \\
\vdots & 0 & 1 & -3 & 1 & 0 & \cdots & 0 & 1 & & & & & & & & & & & & \\
0 & \vdots & 0 & 1 & -2 & 0 & 0 & \cdots & 0 & 1 & & & & & 0 & & & & & & \\
1 & 0 & \vdots & 0 & 0 & -3 & 1 & 0 & \cdots & 0 & 1 & & & & & & & & & & \\
 & 1 & 0 & \vdots & 0 & 1 & -4 & 1 & 0 & \cdots & 0 & 1 & & & & & & & & & \\
 & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & & & \\
 & & & 1 & 0 & \vdots & 0 & 1 & -3 & 0 & 0 & \cdots & 0 & 1 & & & & & & & \\
 & & & & 1 & 0 & \vdots & 0 & 1 & -4 & 1 & 0 & \cdots & 0 & 1 & & & & & & \\
 & & & & & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
 & & & & & & 1 & 0 & \vdots & 0 & 1 & -4 & 1 & 0 & \cdots & 0 & 1 & & & & \\
 & & & & & & & 1 & 0 & \vdots & 0 & 1 & -3 & 0 & 0 & \cdots & 0 & 1 & & & \\
 & & & & & & & & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
 & & & & & & & & & 1 & 0 & \vdots & 0 & 1 & -4 & 1 & 0 & \cdots & 0 & 1 & \\
 & & & & & & & & & & 1 & 0 & \vdots & 0 & 1 & -3 & 0 & 0 & \cdots & 0 & 1 \\
 & & & 0 & & & & & & & & 1 & 0 & \vdots & 0 & 0 & -2 & 1 & 0 & \cdots & 0 \\
 & & & & & & & & & & & & 1 & 0 & \vdots & 0 & 1 & -3 & 1 & 0 & \\
 & & & & & & & & & & & & & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & & & & 1 & 0 & \vdots & 0 & 1 & -3 & 1 \\
 & & & & & & & & & & & & & & & 1 & 0 & \vdots & 0 & 1 & -2
\end{bmatrix}, \tag{7}$$

whose elements are results of several measurement data of $\varphi^{x}_{i,j}$ and $\varphi^{y}_{i,j}$, and $\boldsymbol{A}$ is the coefficient matrix, which is a special sparse $NM \times NM$ ($NM = N \times M$) matrix or

$$\boldsymbol{A} = \begin{bmatrix} A_1 & I & 0 & \cdots & 0 \\ I & A_2 & I & 0 & \cdots \\ 0 & I & \cdots & \cdots & 0 \\ \cdots & 0 & \cdots & \boldsymbol{A}_{N\times M-1} & I \\ 0 & \cdots & 0 & I & \boldsymbol{A}_{N\times M} \end{bmatrix},$$

where $\boldsymbol{I}$ is $N \times M$ unit matrix and $\boldsymbol{A}_1 \cdots \boldsymbol{A}_{N\times M}$ is tri-diagonal matrixes

$$\boldsymbol{A}_1 = \boldsymbol{A}_{N\times M} = \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & -3 & 1 & 0 & \cdots \\ 0 & \cdots & \cdots & \cdots & 0 \\ \cdots & 0 & 1 & -3 & 1 \\ 0 & \cdots & 0 & 1 & -2 \end{bmatrix},$$

$$\boldsymbol{A}_2 = \boldsymbol{A}_3 = \cdots = \boldsymbol{A}_{N\times M-1} = \begin{bmatrix} -3 & 1 & 0 & \cdots & 0 \\ 1 & -4 & 1 & 0 & \cdots \\ 0 & \cdots & \cdots & \cdots & 0 \\ \cdots & 0 & 1 & -4 & 1 \\ 0 & \cdots & 0 & 1 & -3 \end{bmatrix}.$$

Thus the wavefront recovery problem becomes a problem to solve the linear algebraic equation represented by Eq. (6). Usual algorithms such as Gaussian elimination and LU decomposition are apparently not proper when $\boldsymbol{A}$ is so large.

Since the matrix $\boldsymbol{A}$ is a positive definite symmetry matrix, it can be factorized by the Choleski's factorization method for convenient equation solving.

Firstly, the matrix is expressed in a compressed form according to the built relation

$$C_{ij} = \begin{cases} A^{i,\, i=M+j-1}, & i+j-1>0 \\ 0 & i+j-1 \le 0 \end{cases}$$

for $i = 1, 2, \cdots, MN$; $j = 1, 2, \cdots, M+1$

The compressed form is obtained as

$$\boldsymbol{C}_{NM\times(M+1)} = \left[\begin{array}{ccccccccccccccccccccc} -2 & -3 & \cdots & -3 & -2 & -3 & -4 & \cdots & -4 & -3 & \cdots & -3 & -4 & \cdots & -4 & -3 & -2 & -3 & \cdots & -3 & -2 \\ 0 & 1 & 1 & \cdots & 1 & 0 & 1 & \cdots & 1 & 1 & \cdots & \cdots & 1 & \cdots & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & -3 & \cdots & -3 & -2 & -3 & -4 & \cdots & -4 & -3 & \cdots & -3 & -4 & \cdots & -4 & -3 & -2 & -3 & \cdots & -3 & -2 \end{array}\right]^{\mathrm{T}}, \quad (8)$$

Apparently the memory needed is only $1/N$ of that needed by usual solving method.

Secondly, by Choleski method[16], matrix $\boldsymbol{A}$ is factorized as

$$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}'$$

and $\boldsymbol{L}$ is a down-tri matrix which can be obtained based on $\boldsymbol{C}_{NM\times(M+1)}$ and stored in the same matrix space of $\boldsymbol{C}_{NM\times(M+1)}$ just like $\boldsymbol{A}$ in $\boldsymbol{C}_{NM\times(M+1)}$.

According to Choleski factorize equation[16],

$$L_{1,M+1} = \sqrt{C_{1,M+1}}, \quad (9)$$

and the first column of $\boldsymbol{L}$ is

$$L_{jk} = \frac{C_{jk}}{L_{1k}} \text{ for } j = 2, 3, \ldots M+1, \text{ and } k = M-j+2, \quad (10)$$

For $i = 2, 3, \ldots M$, the ith row of $\boldsymbol{L}$ is determined by

$$L_{i,M+1} = \sqrt{C_{i,M+1} - \sum_{k=1}^{M} L_{ik}^{2}}, \quad (11)$$

and for $j = 1, 2, \ldots M$

$$L_{(i+j),n} = \frac{1}{L_{i,m+1}}\left(C_{(i+j),n} - \sum_{k=1}^{n-1} L_{(i+j),k} L_{nk} \right),$$

$$n = M - j + 1, \quad (12)$$

and

$$L_{MN,M+1} = \sqrt{C_{MN,M+1} - \sum_{k=1}^{M} L_{MN,k}^{2}}, \quad (13)$$

Thus the equation (6) becomes

$$\boldsymbol{L}\boldsymbol{L}'\boldsymbol{x} = \boldsymbol{p}, \quad (14)$$

Finally, the solution of Eq. (6) is obtained by solving Eq. (15) as the following steps.

Let $y = \boldsymbol{L}'\boldsymbol{x}$, the equation is firstly solved using the forward substitution,

$$y_1 = \frac{p_1}{L_{1,M+1}}, \quad (15)$$

and for $i = 2, 3, \ldots, MN$

$$y_i = \frac{1}{L_{i,M+1}}\left[p_i - \sum_{k=1}^{M} L_{ik} Y_{i-(M-k+1)}\right]. \quad (16)$$

Then by backward substitution method as the following,

$$x_n = \frac{y_n}{l_{n,M+1}}, \quad (17)$$

and for $i = n-1, n-2, \ldots 1$,

$$x_i = \frac{1}{L_{i,M+1}}\left[y_i - \sum_{k=i+1}^{i+M} L_{k,M+1-k} x_k\right], \quad (18)$$

the recovery result is obtained as

$$\phi_i = x_i,\ i = 1, 2, \cdots, MN. \quad (19)$$

It can be noticed that the memory needed and the calculating quantity in the proposed algorithm are only $2/N$ of that in usual algorithms, and so is the time costing.

3 Noise suppression

Generally it is inevitable that the phase differences obtained by practical shearing interference measurements are with noises, which can cause phase recovery errors.

Assume the noises to be random, additive, and of zero mean and variance of σ, and the noised phase differences are expressed

$$\tilde{\varphi}^x_{ij} = \varphi^x_{ij} + \delta\varphi^x_{ij},$$
$$\tilde{\varphi}^y_{ij} = \varphi^y_{ij} + \delta\varphi^y_{ij},$$

where $\delta\varphi^x_{ij}$, $\delta\varphi^y_{ij}$ are the noise distributions of the measurements, and they are independent and satisfy

$$E(\delta\varphi^x_{ij}) = 0,\ E(\delta\varphi^x_{ij})^2 = \sigma^2$$
$$E(\tilde{\varphi}^y_{ij}) = 0,\ E(\delta\varphi^y_{ij})^2 = \sigma^2$$

Similarly, p in equation (6) can be

$$\tilde{p}_i = p_i + \delta p_i,$$

where $\tilde{p}_i$ is result of obtained phase differences by Eq. (5) and (16).

According to error theory,

$$E(\delta p_i) = 0,$$
$$E(\delta p_i)^2 = E(\delta\varphi^x_{i,j})^2 + E(\delta\varphi^y_{i,j})^2 +$$
$$E(\delta\varphi^x_{i+1,j})^2 + E(\delta\varphi^y_{i,j+1})^2 = L_{i,M+1}\sigma^2, \quad (20)$$

Similarly, by Eq. (16) and Eq. (17),

$$E(\delta y_i) = 0,$$
$$E(\delta y_1)^2 = \frac{1}{L_{1,M+1}^2} E(\delta p_1)^2,$$
$$E(\delta y_2)^2 = \frac{1}{L_{2,M+1}^2}(L_{2M}^2 E(\delta y_1)^2 < E(\delta y_1)^2,$$
$$\ldots\ldots$$
$$E(\delta y_M)^2 = \frac{1}{L_{M,M+1}^2}(L_{MM}^2 E(\delta y_{M-1})^2)$$
$$< E(\delta y_1)^2. \quad (21)$$

For $i > M$

$$E(\delta y_i)^2 = \frac{1}{L_{i,M+1}^2}(E(\delta p_i)^2 + L_{i1}^2 E(\delta y_{i-M})^2 +$$
$$L_{i2}^2 E(\delta y_{i-M-1})^2 + \cdots + L_{iM}^2 E(\delta y_{i-1})^2)$$
$$< \frac{1}{L_{i,M+1}^2}(L_{i,M+1}\sigma^2 + L_{i1}^2 E(\delta y_1)^2 +$$
$$L_{i2}^2 E(\delta y_1)^2 + \cdots + L_{iM}^2 E(\delta y_1)^2)$$
$$= \frac{1}{L_{i,M+1}^2}(L_{i,M+1}\sigma^2 + E(\delta y_1)^2 \sum_{k=1}^{M} L_{i1}^2)$$
$$< \frac{1}{L_{i,M+1}^2}(L_{i,M+1}\sigma^2 + E(\delta y_1)^2) =$$
$$\left(\frac{1}{L_{i,M=1}^3} + \frac{1}{L_{i,M=1}}\right)\sigma^2 < 2\sigma^2, \quad (22)$$

Then by Eq. (18), Eq. (19) and Eq. (20)

$$E(\delta x_i) = 0,$$
$$E(\delta x_{MN})^2 = \frac{1}{L_{MN,M+1}^2} E(\delta y_{MN})^2 < 2\sigma^2$$
$$E(\delta x_{MN-1})^2 = \frac{1}{L_{MN-1,M+1}^2}(E(\delta y_{MN})^2 +$$
$$L_{MN,M}^2 E(\delta x_{MN})^2)$$
$$= \frac{1}{L_{MN-1,M+1}^2}(E(\delta y_{MN})^2 +$$
$$L_{MN,M}^2 \times \frac{1}{M_{MN,M}} E(\delta y_{MN})^2) < 4\sigma^2$$
$$\ldots\ldots$$
$$E(\delta x_1)^2 = \frac{1}{L_{1,M+1}^2}(E(\delta y_1)^2 + L_{M,1}^2 E(\delta x_M)^2 +$$
$$L_{M-1,1}^2 E(\delta x_{M-1})^2 + \cdots +$$
$$L_{2,1}^2 E(\delta x_2)^2) < E(\delta y_1)^2$$

and

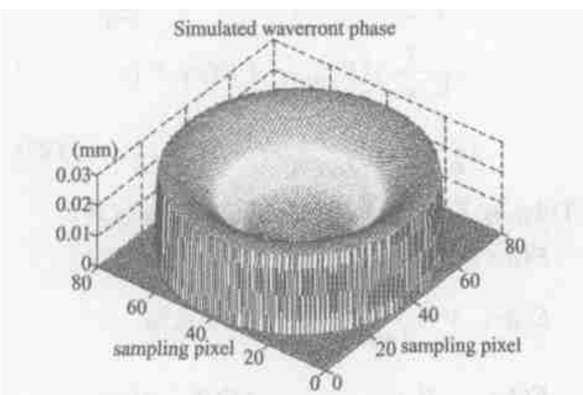
$$E(\delta x_i)^2 = \frac{1}{L_{i,M+1}^2}(E(\delta y_i)^2 +$$
$$L_{i+1,M-i}^2 E(\delta x_{i+1})^2 +$$

$$L_{i+2,M-i-1}{}^2E(\delta x_{i+2})^2+\cdots+L_{i+M,1-i}{}^2E(\delta x_{i-1})^2)$$
$$<\frac{1}{L_{i,M+1}{}^2}(L_{i,M+1}\sigma^2+L_{i1}{}^2E(\delta y_1)^2+L_{i2}{}^2E(\delta y_1)^2+\cdots+L_{iM}{}^2E(\delta y_1)^2)$$
$$=\frac{1}{L_{i,M+1}{}^2}(L_{i,M+1}\sigma^2+E(\delta y_1)^2\sum_{k=1}^{M}L_{i1}{}^2)$$
$$<\frac{1}{L_{i,M=1}{}^2}(L_{i,M+1}\sigma^2+E(\delta y_1)^2)=$$
$$(\frac{1}{L_{i,M=1}{}^3}+\frac{1}{L_{i,M=1}})\sigma^2<\frac{5}{8}\sigma^2\quad,\tag{23}$$

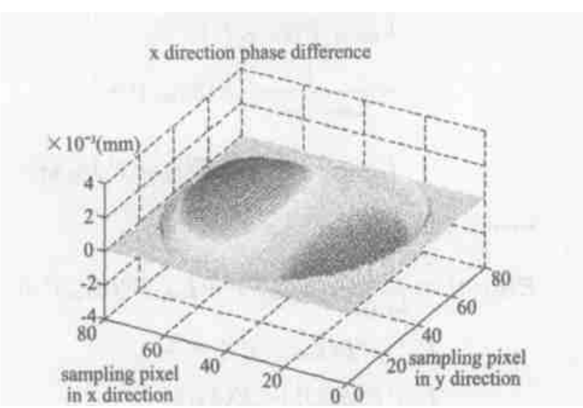
So by the proposed algorithm, the mean square error due to noise for every pixel of the recovery result is less than 2σ. The error suppression effect of the algorithm is shown.

4 Simulation

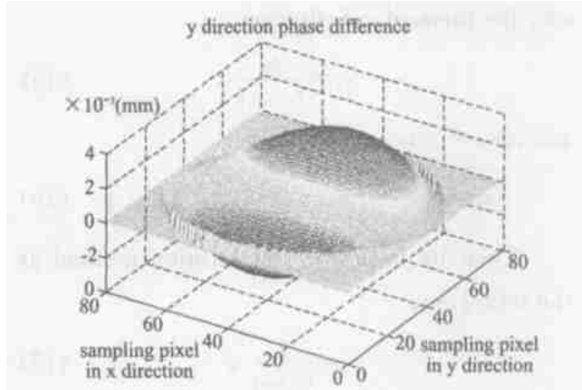
To prove the feasibility and the advanced properties of the proposed algorithm, computer simulations and experiments are carried out as follows: (1) Simulate an arbitrary wavefront phase; (2) find its phase differences in x and y directions; (3) recover the wavefront phase from the phase differences by the proposed algorithm; (4)



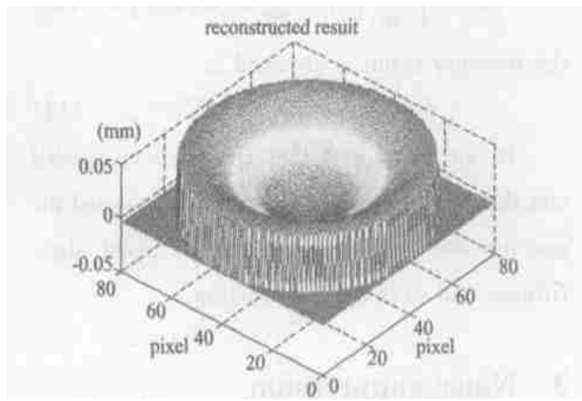
(a) Simulated wavefront phase



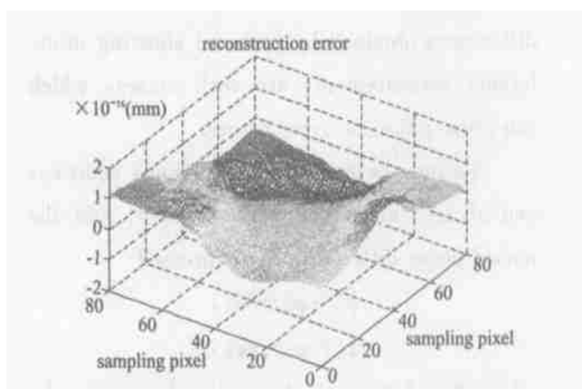
(b) Phase difference along x- direction;



(c) Phase difference along y- direction



(d) Recovery result by proposed algorithm

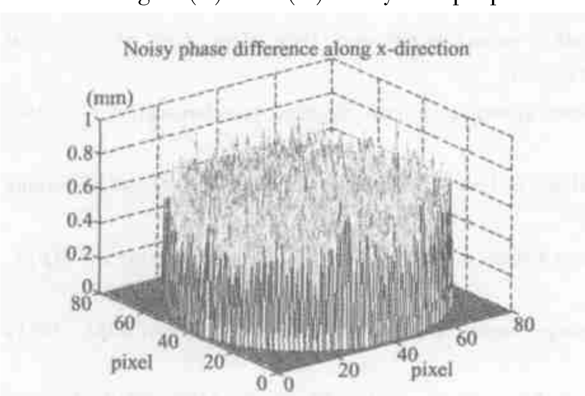


(e) Result error compared with the original wavefront phase

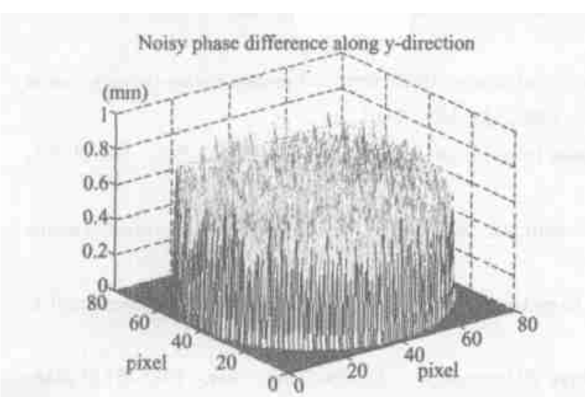
Fig. 1　Computer testing of the proposed algorithm compare the result with the simulated wavefront phase to evaluate the recovery accuracy. To evaluate the noise suppression property of the algorithm, random noises are added to the phase differences, then the wavefront phases are recovered from the noised differences and compared with the original wavefront phase, the impact of noise is shown.

The simulated test wavefront phase is shown in Fig. 1 (a). According to shearing interference principle, the phase differences along x and y direction are shown in Fig. 1 (b) and (c), respectively. Apply the proposed phase recovery algorithm to the two groups of differences, the recovery result is shown in Fig. 1 (d). Fig. 1 (e) shows the recovery error, the peak-valley (PV) value for the error doesn't exceed 2×10^{-16} mm.

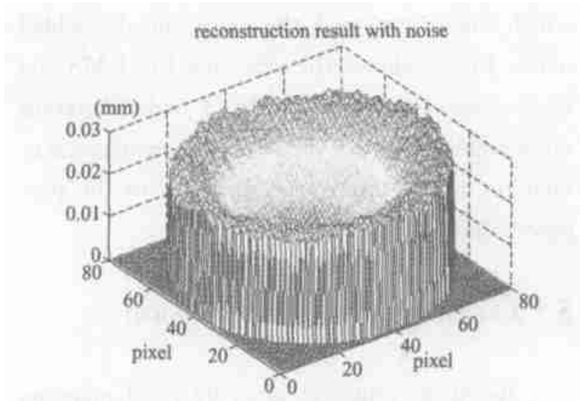
Random Gaussian noises, whose means are 0 and variance $\sigma = 0.01$ mm, are added to the phase differences to simulate noisy phase differences shown in Fig. 2(a) and (b). By the proposed



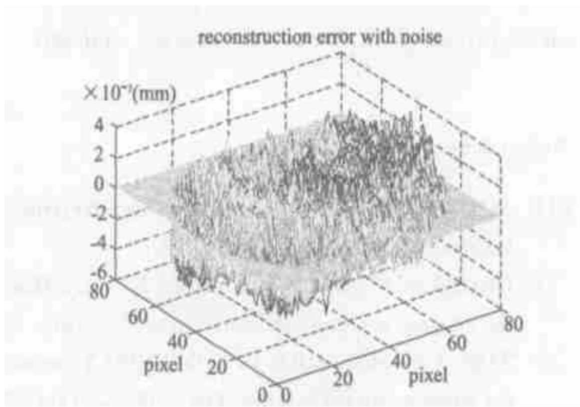
(a) Noisy phase differences along x-direction



(b) Noisy phase differences along y-direction



(c) Recovery result



(d) Recovery error compared with the original phase with noise

Fig. 2　Recovery-result with noise, $\sigma = 0.01$ mm

algorithm, the corresponding phase recovery result is obtained and shown in Fig. 2(c).

The result error comparing with the simulated wavefront phase is shown in Fig. 2 (d). The mean-square-root (RMS) error is 0.001 2 mm,

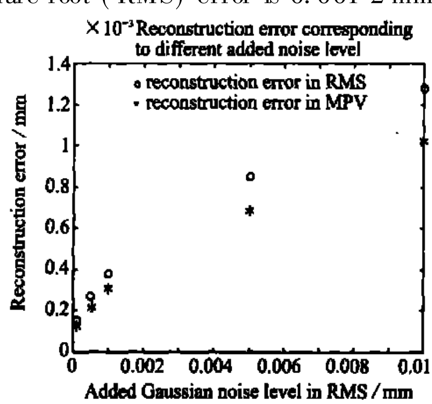


Fig. 3　Error level (RMS) under different noise conditions (variance σ).

which doesn't exceed the range of the added noise. Fig. 3 shows the error level in RMS and mean-peak-valley height (MPV) under different noise conditions (variance σ). The conclusion about the noise suppression property of the proposed algorithm is proved.

5 Discussion and conclusion

Based on complete pixel by pixel mapping relationship between test phase and its differences, together with the least square principle, an algorithm with improved accuracy and efficiency is proposed for test wavefront phase recovery from obtained phase differences in shearing interferometry. In the algorithm, a special linear equation set is firstly built, and special solving method for the linear equation is adopted to reduce time cost and computer memory need and improve accuracy and efficiency. The excellent characteristics of the algorithm have been demonstrated by computational and experimental testing on the proposed algorithm and its comparison with others. Good noise suppression ability is proved by error propagation characteristic analysis.

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第三届全国信息获取与处理学术会议
征 文 通 知

第三届全国信息获取与处理学术会议将于2005年8月在浙江省金华市召开,本次会议由中国仪器仪表学会主办,沈阳市仪器仪表与自动化学会和浙江师范大学联合承办。

一、征文范围

1.各种电量和非电量的测量方法(如机械量、热工量、物性和化学成分量、状态量等);2.各种测量装置(如传感器、敏感元件、敏感材料、测量电路、处理电路、显示电路等);3.测量过程中的信号传输(如现场总线等);4.模式识别(如图像、语音等);5.误差与数据处理;6.故障诊断;7.过程控制方法与装置;8.其它。

二、征文要求

1.论文未公开发表;2.来稿请严格按照《沈阳测控信息网》提供的排版格式要求排版,打印两份,包括中英文摘要,参考文献,全文原则上控制在二个版面,摘要的撰写请参考《Ei数据库文摘要求》;3.受各类基金资助的论文,请注明项目名称及编号;4.请用A4纸打印论文题目,作者姓名,简历,单位,通讯地址,邮编,电话,传呼,手机,E-mail等;5.来稿请通过电子邮件和邮局同时投寄;6.版权纠纷,作者自负。

三、重要信息

本次会议将评选优秀论文,由中国仪器仪表学会颁发优秀论文证书,并在《沈阳测控信息网》公布获奖名单。

本次会议录用论文将刊登在代表中国仪器仪表领域最高学术水平的学术性刊物,《仪器仪表学报》2005年第3期增刊上。《仪器仪表学报》增刊从2003年起已作为Ei Page One收录源,目前被《中国学术期刊网》、《万方数据网》等著名检索机构全文收录。

为便于交流,在本次会议的资料汇编中将刊登参会作者简介及通讯方式。

有关会议具体时间,日程安排、金华概况等会议相关信息请关注《沈阳测控信息网》。

四、重要日期

论文截稿日期为2005年3月15日,论文录用通知发出日期为2005年4月1日。

五、投稿地址

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中国仪器仪表学会
2004年**9**月**20**日